

Probing the symmetry energy and the degree of isospin equilibrium

Qingfeng Li^{1*}, Zhuxia Li², and Horst Stöcker^{1,3}

1) Frankfurt Institute for Advanced Studies (FIAS),

Johann Wolfgang Goethe-Universität,

Max-von-Laue-Str. 1,

D-60438 Frankfurt am Main, Germany

2) China Institute of Atomic Energy,

P.O. Box 275 (18),

Beijing 102413, P.R. China

3) Institut für Theoretische Physik,

Johann Wolfgang Goethe-Universität,

Max-von-Laue-Str. 1,

D-60438 Frankfurt am Main, Germany

* Fellow of the Alexander von Humboldt Foundation.

Abstract

The rapidity dependence of the single- and double- neutron to proton ratios of nucleon emission from isospin-asymmetric but mass-symmetric reactions $\text{Zr}+\text{Ru}$ and $\text{Ru}+\text{Zr}$ at energy range $100 \sim 800$ A MeV and impact parameter range $0 \sim 8$ fm is investigated. The reaction system with isospin-asymmetry and mass-symmetry has the advantage of simultaneously showing up the dependence on the symmetry energy and the degree of the isospin equilibrium. We find that the beam energy- and the impact parameter dependence of the slope parameter of the double neutron to proton ratio (F_D) as function of rapidity are quite sensitive to the density dependence of symmetry energy, especially at energies $E_b \sim 400$ A MeV and reduced impact parameters around 0.5. Here the symmetry energy effect on the F_D is enhanced, as compared to the single neutron to proton ratio. The degree of the equilibrium with respect to isospin (isospin mixing) in terms of the F_D is addressed and its dependence on the symmetry energy is also discussed.

PACS numbers: 24.10.Lx, 25.75.Dw, 25.75.-q

High energy collision of heavy ions provides a unique tool to study the unbroken properties of dense nuclear matter at finite temperature, outside of the reach of conventioned nuclear physics [1, 2]. The FOPI Collaboration has recently performed the mixing experiment by using four mass 96+96 systems $^{96}_{44}\text{Ru}+^{96}_{44}\text{Ru}$, $^{96}_{40}\text{Zr}+^{96}_{40}\text{Zr}$, $^{96}_{44}\text{Ru}+^{96}_{40}\text{Zr}$, and $^{96}_{40}\text{Zr}+^{96}_{44}\text{Ru}$ to investigate the degrees of the isospin-equilibrium in the intermediate energy heavy ion collisions (HICs) [3, 4]. It was found that complete isospin equilibration is not reached by comparing the emitted proton number counted in rapidity (see [3, 4, 5, 6, 7]). The advantage of taking mixed reactions is that the isospin asymmetry shows up more obviously because the effects of the iso-scalar part of the equation of state (EoS) largely cancel. This advantage could also be used for probing the density dependence of the symmetry energy. So far the density dependence of the nuclear symmetry energy, $E_{\text{sym}}(u)$, with $u = \rho/\rho_0$ being the reduced nuclear density, is still largely uncertain, although a lot of theoretical and experimental efforts have been undertaken in the past decade [8, 9, 10]. Recently, the symmetry energy has been extracted to $E_{\text{sym}}(u) \simeq 31.6u^{1.05}$ at subnormal densities by comparing with experimental data [11] when a free nucleon-nucleon (NN) elastic cross section has been used [12], while $E_{\text{sym}}(u) \simeq 31.6u^{0.69}$ after considering partly medium modifications of the nucleon transport [13, 14] were subtracted. The symmetry energy can be investigated by the double neutron to proton and π^-/π^+ ratios. The neutron-rich $^{124,132}\text{Sn} + ^{124}\text{Sn}$ system and more isospin-symmetric $^{112}\text{Sn} + ^{112}\text{Sn}$ system serve as robust probes of the nuclear symmetry energy [15, 16]. A very attractive feature of taking double ratios is the fact that the systematic errors can be strongly reduced, while the sensitivity of the symmetry energy is close to the single neutron to proton and π^-/π^+ ratios.

In the present work, we take two sets of isospin-asymmetric but mass-symmetric reactions, $\text{Zr}+\text{Ru}$ and $\text{Ru}+\text{Zr}$, to study the symmetry energy effect on the rapidity dependence of both, single as well as double unbound neutron to proton ratios. We take advantage of the mixing reactions to study the influence of the isospin dependent part of the EoS on the equilibration process. We show that the slope of the double neutron to proton ratio as distributed in the rapidity space is — for mixing reactions — closely related to degree of isospin equilibration. The UrQMD transport model has been updated for simulating intermediate energy HICs [2, 17, 18, 19, 20, 21]. It is also used for the calculations presented in this work. A soft equation of state (S-EoS) is adopted [20] (except otherwise stated). We consider here two sets of symmetry energy parameterizations: $E_{\text{sym}}(u) = 34u^\gamma$ with $\gamma = 0.5$ (soft) and 1.5

(hard). A non-relativistic medium modification of the nucleon-nucleon elastic cross section is also considered here (please see Ref. [21] for details). The method to construct the freeze-out clusters is the same as presented in our previous work [19, 21].

We calculate the rapidity ($Y_{cm}^{(0)} = y_{cm}/y_{beam}$ in the center-of-mass system) dependence of unbound neutrons and protons in Zr+Ru and Ru+Zr reactions at the beam energies $E_b = 100, 200, 400, 600,$ and 800 A MeV and at the reduced impact parameters $b/b_0 = 0, 0.25, 0.5,$ and 0.75 , where $b_0 = R_{proj} + R_{targ}$ is the maximum impact parameter leading to nuclear reactions by adopting 'sharp-cutoff' approximation. The calculated rapidity dependence of the neutron to proton " n/p " ratios for Zr+Ru and Ru+Zr reactions are shown in Fig. 1 (a)-(d) for $E_b = 100$ A MeV, $b/b_0 = 0.5$ in plot (a); $E_b = 400$ A MeV, $b/b_0 = 0.5$ in (b); $E_b = 400$ A MeV, $b/b_0 = 0$ in (c); and $E_b = 800$ A MeV, $b/b_0 = 0$ in (d). If equilibrium with respect to the isospin degree of freedom was reached, the rapidity distributions of the neutron to proton ratios for the two different mixing reactions should coincide with each other. Figs. 1 (a) and (b) show clearly that for $b/b_0 = 0.5$ equilibrium (with respect to isospin degree of freedom) is not reached, because the centroid of the rapidity dependence of the neutron to proton ratio is not located at mid-rapidity [5]. For Zr+Ru reactions, the neutron to proton ratio distribution peaks at the positive rapidity, while for the inverse Ru+Zr reactions, the distribution peaks at the negative rapidity. The rapidity dependence of the neutron to proton ratio is symmetric with respect to $Y_{cm}^{(0)} = 0$ for central reactions at high beam energy (Fig. 1 (d)), where the equilibration is nearly reached. It is interesting to see that the statistics in the UrQMD calculations is perfectly guaranteed since the calculated results for Zr+Ru and Ru+Zr reactions are symmetrically distributed very well with respect to $Y_{cm}^{(0)} = 0$, as one expects. This is also a fundamental but important checkup in the experimental side. As seen for reactions at $E_b = 400$ A MeV and $b/b_0 = 0$ in Fig. 1 (c), the rapidity distribution of the neutron to proton ratio almost coincides for reactions Zr+Ru and Ru+Zr when the soft-symmetry energy is chosen, but it deviates strongly when the stiff-symmetry energy is chosen. This shows the closely relationship between the effect of the symmetry energy and the equilibration in HICs at intermediate energies.

Figs. 1 (e) and (f) present the corresponding rapidity distributions of double neutron to proton ratio " $(n/p)_{ZrRu}/(n/p)_{RuZr}$ " with the same conditions as Figs. 1 (a) and (d), respectively. From Fig. 1 (e) it is seen that the double ratio increases almost linearly from target ($Y_{cm}^{(0)} = -1$) to projectile ($Y_{cm}^{(0)} = 1$) rapidity region for the case of $E_b=100$ A MeV and

$b/b_0 = 0.5$. The two curves corresponding to soft and stiff symmetry energies cross at the point of rapidity equal to zero, with positive but different slopes. And obviously, the slope for $\gamma = 0.5$ case is larger than that for $\gamma = 1.5$ case. It is clear that for isospin asymmetric and mass symmetric reactions Zr+Ru and Ru+Zr, if the equilibrium with respect to isospin degree of freedom is totally reached, the double ratio should be unity at all rapidities. The deviation of $(n/p)_{Zr+Ru}/(n/p)_{Ru+Zr}$ from unit thus implies the deviation from isospin equilibration. Fig. 1 (e) shows clearly that the isospin equilibration is not reached and the choice of symmetry energy influences the isospin equilibrium process for this case. While for the reactions at $E_b = 800$ A MeV and $b/b_0 = 0$ presented in Fig. 1 (f), the slope is ~ 0 and becomes horizontal in large rapidity region, which means that the isospin equilibration can be roughly reached. Thus the double neutron to proton ratio as function of rapidity taking from the mixing reaction system can provide both the information of density dependence of the symmetry energy and the degree of isospin equilibrium reached in the reaction.

Further, we study the beam energy and the centrality dependence of the slope parameters of the double neutron to proton ratio over the rapidity, namely the F_D , which is obtained by linearly fitting the rapidity distribution of the double ratio over $|Y_{cm}^{(0)}| \leq 1$. Fig. 2 (a) shows the excitation function of F_D for impact parameters $b/b_0 = 0$ and 0.5 and for symmetry energies $\gamma = 0.5$ and 1.5 , respectively. The reactions with larger impact parameters show larger F_D values, which implies that less equilibration is reached. The F_D values first rise and then drop slightly with increasing beam energy. It shows a broad maximum at $E_b \sim 200-400$ A MeV for intermediate impact parameters, while central collisions peak at 200 A MeV. The influence of the symmetry energy on the F_D is stronger for $b/b_0 = 0.5$ than for $b/b_0 = 0$. The isoscalar part of the EoS affects the equilibrium process of the colliding system as well [5]. Fig. 2 (a) shows F_D at $E_b = 400$ A MeV and $b/b_0 = 0.5$ using a soft EoS with momentum dependence (SM-EoS, open dots). It is seen that the difference between the S-EoS and the SM-EoS is very small. This indicates that the effect of the isoscalar part of the EoS on F_D is largely cancelled, while the effect of the isovector part on F_D is nicely preserved.

Fig. 2 (a) also shows that — for central collisions — F_D as calculated with a stiff symmetry energy is larger than with a soft one. For $b/b_0 = 0.5$, the relative magnitude of F_D as calculated with the stiff symmetry energy becomes smaller than that with the soft symmetry energy. The centrality dependence (with $b/b_0 = 0, 0.25, 0.5$, and 0.75) of F_D at beam energy 400 A MeV is presented in Fig. 2 (b). This shows that both curves of F_D , as calculated with

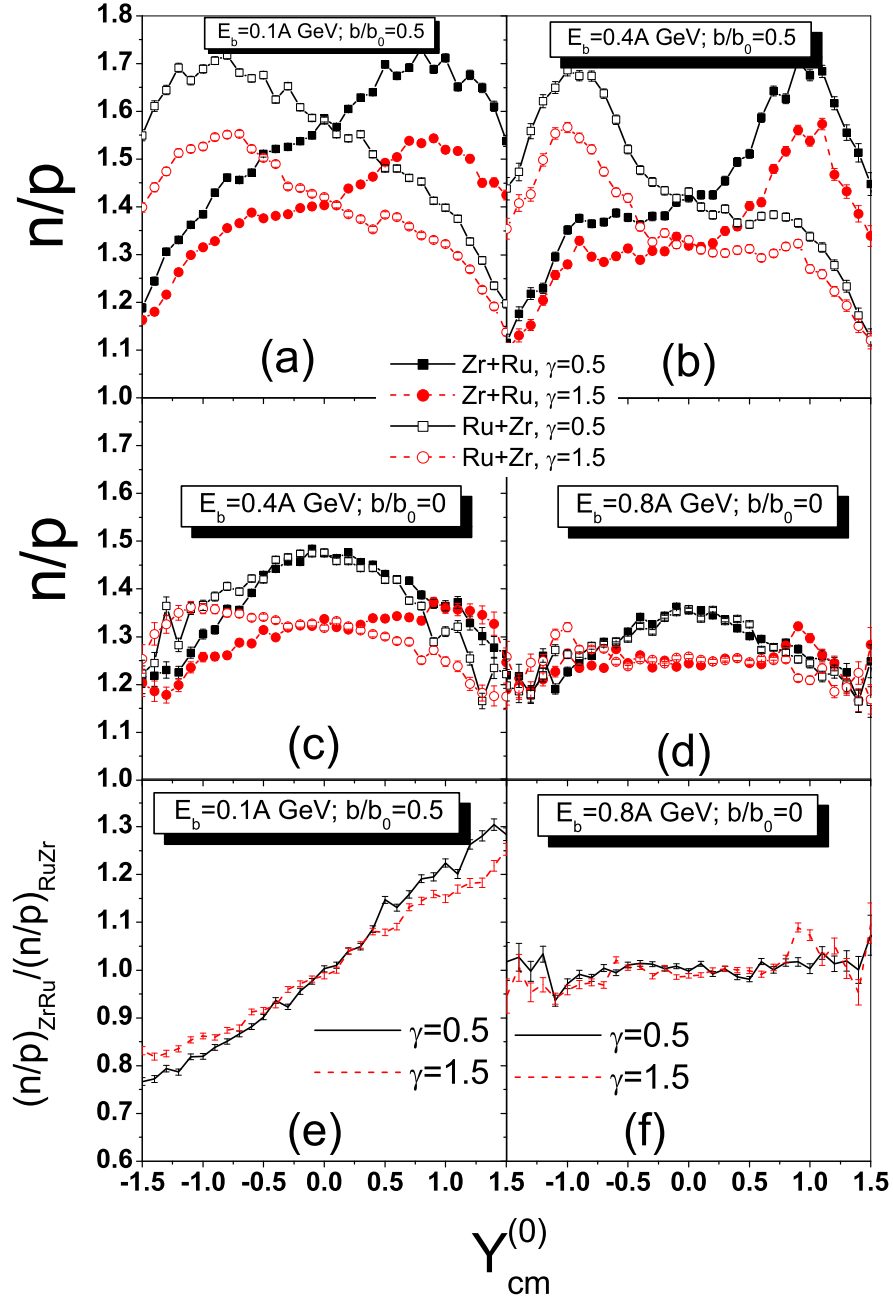


FIG. 1: (Color online) (a)-(d): Rapidity dependence of neutron to proton (n/p) ratios with symmetry energies of $\gamma = 0.5$ and 1.5 . The Zr+Ru and Ru+Zr reactions with $E_b = 100$ A MeV, $b/b_0 = 0.5$ in plot (a), $E_b = 400$ A MeV, $b/b_0 = 0.5$ in (b), $E_b = 400$ A MeV, $b/b_0 = 0$ in (c), and $E_b = 800$ A MeV, $b/b_0 = 0$ in (d) are chosen. (e)-(f): the corresponding double neutron to proton ratios $((n/p)_{ZrRu}/(n/p)_{RuZr})$ for (a) and (d) cases (see text).

soft and stiff symmetry energies, increase monotonously with impact parameter. The F_D value obtained with the stiff symmetry energy is larger than that with the soft symmetry energy, when $b/b_0 \lesssim 0.2$. And they reverse when $b/b_0 \gtrsim 0.2$. The reversal of the relative magnitudes of F_D with stiff or soft symmetry energy at small and large impact parameters is due to the gradual change of the mechanism of particle emission [22]. Central collisions are more violent, hence the compression is stronger than for the collisions with large impact parameters. Correspondingly, the degree of isospin equilibration is both higher and reached more quickly. Nucleons emitted from central collisions stem mainly from the mid-rapidity region, where matter is much more compressed than at the projectile/target region. During the compression phase, the stiff symmetry potential provides stronger repulsion (attraction) than the soft one for neutrons (protons) in the neutron-rich matter. Therefore, the center of collisions is more strongly compressed. However, higher pressure caused by the stiff symmetry potential leads to faster emission of particles. Hence, fewer two-body collisions occur and the equilibration decreases. For peripheral reaction, the average compression is relatively weak, Nucleon emission takes place mainly at projectile/target rapidity, where the nuclear density is low. The low-density nuclear surface thus plays an important role for nucleon emission. At low densities, the stiff symmetry potential provides weaker repulsion (attraction) on neutrons (protons) than the soft one in the neutron-rich nuclear medium. Higher densities result in the projectile/target region, leading to an enhanced number of two-body collisions. The smaller F_D value is evident with the stiff symmetry energy.

The effect of the symmetry energy on F_D becomes the most pronounced at $E_b \sim 400$ A MeV and at large impact parameters (Fig. 2). Compare the isospin effects in Fig. 1 (b) and Fig. 2 for reactions at $E_b = 400$ A MeV and $b/b_0 = 0.5$: The double neutron to proton ratio exhibits an enhanced sensitivity ($\sim 30\%$) to the density dependence of the symmetry energy as compared to the single neutron to proton ratio ($\sim 10\%$). Thus, the double neutron to proton ratio (taken from mass symmetric, but isospin asymmetric mixing systems at energies around 400 A MeV and at semi-peripheral impact parameters) is a prominent probe of the density dependence of the symmetry energy.

In summary, we have investigated the rapidity dependence of the single- and double neutron to proton ratios as taken from isospin-asymmetric but mass-symmetric systems Zr+Ru and Ru+Zr. The beam energy- and the centrality dependence of the slope parameters of the double neutron to proton ratio (F_D) as function of rapidity are presented. We find

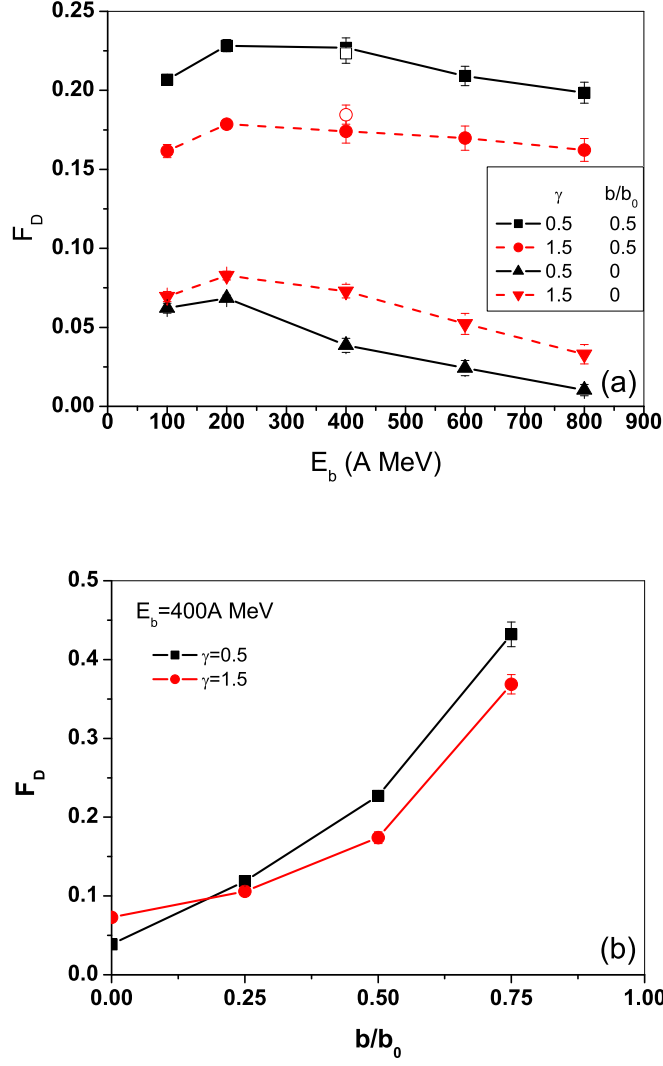


FIG. 2: (Color online) (a): Beam energy dependence of the slope parameter F_D with symmetry energies with $\gamma = 0.5$ and 1.5 , for impact parameters $b/b_0 = 0$ and 0.5 . The F_D values at $E_b = 400$ A MeV and at $b/b_0 = 0.5$ with SM-EoS are shown with open dots. (see text) (b): The centrality dependence of F_D at $E_b = 400$ A MeV.

that both, the rapidity distribution of the single neutron to proton ratio and the F_D are sensitive to the density dependence of symmetry energy. The study of the beam energy- and the centrality dependence of the F_D values shows that the (semi-) peripheral HICs at energies around $E_b \simeq 400$ A MeV are most suited for probing the density dependence of

the symmetry energy. Here the symmetry energy effect is enhanced most strongly as is observable in the rapidity dependence of the double neutron to proton ratio, as compared to the single ratio.

Acknowledgments

Q. Li thanks the Alexander von Humboldt-Stiftung for a fellowship. This work is partly supported by the National Natural Science Foundation of China under Grant No. 10235030 and the Major State Basic Research Development Program of China under Contract No. G20000774, as well as by GSI, BMBF, DFG, and Volkswagenstiftung. We gratefully acknowledge support by the Frankfurt Center for Scientific Computing (CSC).

-
- [1] H. Stöcker and W. Greiner, Phys. Rept. **137** (1986) 277.
 - [2] S. A. Bass *et al.*, Prog. Part. Nucl. Phys. **41**, 225 (1998)
 - [3] F. Rami *et al.* [FOPI Collaboration], Phys. Rev. Lett. **84**, 1120 (2000)
 - [4] B. Hong *et al.*, [FOPI Collaboration], Phys. Rev. C **66**, 034901 (2002).
 - [5] A. Hombach, W. Cassing and U. Mosel, Eur. Phys. J. A **5**, 77 (1999)
 - [6] Q. Li and Z. Li, Phys. Rev. C **64**, 064612 (2001)
 - [7] T. Gaitanos, M. Colonna, M. Di Toro and H. H. Wolter, Phys. Lett. B **595**, 209 (2004)
 - [8] B. A. Li, C. M. Ko and W. Bauer, Int. J. Mod. Phys. E **7**, 147 (1998)
 - [9] *Isospin Physics in Heavy Ion Collisions at Intermediate Energies*, edited by Bao-An Li and W. Udo Schroeder, NOVA Science Publishers, Inc., New York, 2001
 - [10] V. Baran, M. Colonna, V. Greco, M. DiToro, Phys. Rep. **410**, 235 (2005)
 - [11] M. B. Tsang, *et al.*, Phys. Rev. Lett. **92**, 062701 (2004)
 - [12] L. W. Chen, C. M. Ko and B. A. Li, Phys. Rev. Lett. **94**, 032701 (2005)
 - [13] B. A. Li and L. W. Chen, Phys. Rev. C **72**, 064611 (2005)
 - [14] B. A. Li, L. W. Chen, C. M. Ko and A. W. Steiner, arXiv:nucl-th/0601028.
 - [15] B. A. Li, L. W. Chen, G. C. Yong and W. Zuo, Phys. Lett. B **634**, 378 (2006)
 - [16] G. C. Yong, B. A. Li, L. W. Chen and W. Zuo, Phys. Rev. C **73**, 034603 (2006)
 - [17] H. Weber, E. L. Bratkovskaya and H. Stöcker, Phys. Lett. B **545**, 285 (2002)

- [18] Q. Li, Z. Li, S. Soff, R. K. Gupta, M. Bleicher and H. Stöcker, J. Phys. G: Nucl. Part. Phys. **31**, 1359 (2005)
- [19] Q. Li, Z. Li, S. Soff, M. Bleicher and H. Stöcker, Phys. Rev. C **72**, 034613 (2005)
- [20] Q. Li, Z. Li, S. Soff, M. Bleicher and H. Stöcker, J. Phys. G: Nucl. Part. Phys. **32**, 151 (2006)
- [21] Q. Li, Z. Li, S. Soff, M. Bleicher and H. Stöcker, J. Phys. G **32**, 407 (2006)
- [22] Y. Zhang and Z. Li, Phys. Rev. C **71**, 024604 (2005).